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New evidence for 'confined coherence' in weakly coupled Luttinger liquids

David G Clarke†§ and S P Strong‡‖

† IRC in Superconductivity and Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, UK

‡ NEC Research Institute, 4 Independence Way, Princeton, NJ 08540, USA

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Abstract. On the basis of a calculation of the exact inter-liquid hopping rate and an approximate single-particle Green's function, we present new evidence for the existence of a phase of *relevant but incoherent* inter-Luttinger-liquid transport. This phase of 'confined coherence' occurs when the Luttinger liquid exponent α satisfies $\alpha_c < \alpha < 1/2$. We argue that α_c is strictly bounded above by $1/4$, and is probably substantially smaller, especially in spin-charge-separated Luttinger liquids. We also discuss connections with the work of others.

1. Introduction

The physics of a strongly correlated, highly anisotropic electron system represents a subtle problem in many-body physics. In previous work [1, 2] we have considered the problem of one-dimensional (1D) electron liquids coupled by weak inter-liquid hopping. Implicit in our approach is the recognition that if one begins with a collection of truly 1D metals and then turns on weak inter-liquid hopping, it is not *a priori* appropriate to consider the electron–electron interaction as a perturbation on an anisotropic (2D) free Fermi gas. Rather, one should consider the *inter-liquid hopping* as a perturbation on the (otherwise decoupled) 1D liquids. The problem is non-trivial but tractable to some degree, because the low-energy physics of a 1D metal is described by Luttinger liquid theory. Unlike in a Fermi liquid, where the electron spectral function, $\rho(k,\omega)$ is dominated by a quasiparticle part, which sharpens up to a δ-function as $k \to k_F$, in a Luttinger liquid there are no Landau quasiparticles; rather, $\rho(k,\omega)$ exhibits only power-law singularities. For this reason, and others that we have previously discussed [1, 2], the problem of weakly coupled Luttinger liquids is closely analogous to that of weak tunnelling in a two-level system (TLS) coupled to an ohmic dissipative bath [3]. Exploiting this analogy led us to propose [1, 2] that inter-liquid hopping between non-Fermi liquids may have three qualitatively distinct regimes: it may be irrelevant, relevant and coherent, or relevant but entirely incoherent. The incoherent inter-liquid hopping phase would represent a new state of matter with intrinsically incoherent transport in at least one direction. There is substantial experimental support for this proposal based on its ability to explain certain anomalous properties of the low-dimensional organic conductor $(TMTSF)_2PF_6$, as has been discussed elsewhere [4–6]. In this paper we briefly report new results which address this question based upon the use of exact Luttinger liquid

§ Present address: Department of Physics, Princeton University, Princeton, NJ 08544-0708, USA.

‖ Present address: Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, USA.

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spectral functions, careful consideration of the analytic properties of Luttinger liquid Green's functions, and a reinterpretation of a calculation made by others [7].

2. The inter-liquid hopping rate for weakly coupled Luttinger liquids

We are interested in the problem of N coupled Luttinger liquids, $N \to \infty$. At $O(t_\perp^2)$, however, our results are equivalent to those for $N = 2$, and we therefore consider the problem of two Luttinger liquids coupled by a spatially uniform, single-particle hopping (as in [1]). Our calculation is dynamical and involves taking a $t = 0$ state with ΔN more (right-moving) particles in liquid 2 than in liquid 1 and no Tomonaga bosons excited in either, then turning on $t_\perp$ and examining the time dependence of ΔN (for the motivation see [1] and [3]). The particle number difference ΔN entails a Fermi momentum difference Δk and a chemical potential difference $\Delta\mu \equiv v\,\Delta k$. Unlike in our earlier work [1] based upon space-time Green's functions, we use spectral function methods here, which is both physically more illuminating and permits the calculation of key correlation functions *exactly*.

At $O(t_\perp^2)$ the *inter-liquid hopping rate* $\Gamma(t)$ can be written in a spectral function form as

$$\Gamma(t) = 2t_\perp^2 L \int \frac{d\omega}{2\pi}\,\frac{\sin\omega t}{\omega}\{A_{12}(\omega) + A_{21}(\omega)\} \qquad (1)$$

where

$$A_{ij}(\omega) = \int \frac{d\omega'}{2\pi}\int \frac{dk}{2\pi}\;\mathcal{J}_1^{(i)}(k,\omega')\mathcal{J}_2^{(j)}(k,\omega'-\omega) \qquad (2)$$

and the spectral functions $\mathcal{J}_{1,2}(k,\omega)$ are the Fourier transforms of

$$\mathcal{J}_1(k,t) \equiv \langle c_1(k,t)c_1^\dagger(k,0)\rangle \qquad \text{and} \qquad \mathcal{J}_2(k,t) \equiv \langle c_2^\dagger(k,0)c_2(k,t)\rangle.$$

In this paper we consider only the zero-temperature limit, in which case

$$\mathcal{J}_{1,2}(k,\omega') = \theta_\pm(\omega'-\mu)\rho_{1,2}(k,\omega'-\mu)$$

where $\rho(k,\omega)$ is the electron spectral function as conventionally defined. We remark that equations (1) and (2) are not specific to coupled 1D liquids: they may be extended to the case of coupled 2D liquids by replacing k by $\boldsymbol{k}$ in the k-integrals and in the definitions of $\mathcal{J}_1$ and $\mathcal{J}_2$.

Physically, $A_{12}(\omega)$ is the effective inter-liquid hopping spectral function for an electron hopping *to* liquid 1, *from* liquid 2, and $A_{21}(\omega)$ that for the opposite case. As $A_{21}(\omega)$ never has a coherent component, it suffices, for the purposes of studying the question of coherence, to consider just $A_{12}(\omega)$.

Before presenting the calculation of $\Gamma(t)$ for coupled Luttinger liquids, we first show how the coherence of inter-liquid hopping manifests itself in the case of coupled (Landau) Fermi liquids.

2.1. Free Fermi gases, and Fermi liquids

For free Fermi gases, $A_{12}(\omega) \propto \Delta\mu\,\delta(\omega)$ and $A_{21}(\omega) = 0$. Thus $\Gamma(t) \propto \Delta\mu\ t$, a clear signal of coherent hopping and hence of a fundamental rearrangement of the ground state.

In a Fermi liquid the (retarded) Green's function is $G_R^{-1}(k,\omega) = Z^{-1}(\omega - E_k) + i\gamma\omega^2$ where Z is the quasiparticle renormalization factor, and γ is a (positive) parameter characterizing the strength of the electron–electron interactions. The spectral function is then given by $\rho(k,\omega) = -2\,\mathrm{Im}\,G_R(k,\omega)$, from which we obtain

$$A_{12}(\omega) \sim v_F^{-1}\{Z^2\,\Delta\mu\,\delta(\omega) + (3\pi)^{-1}Z^3\gamma\omega\}\,\theta_+(\omega+\Delta\mu). \qquad (3)$$

We find that $\Gamma_{12}(t)$ is a sum of a term $\propto Z^2\,\Delta\mu\, t$ representing fundamentally coherent processes, and a term $\propto\gamma Z^3 t^{-1}$ which is marginal. By choosing a sufficiently small $t_\perp$ one can find a time t such that, while remaining in the perturbative regime, $N^{-1}\int_0^t \Gamma(t')\,\mathrm{d}t' \ll 1$, the ratio of the coherent contribution to the marginal contribution is arbitrarily large. This is true *regardless of how small Z is*. Thus, a perturbative calculation in $t_\perp$ does not reveal any likelihood of a loss of coherence of inter-liquid tunnelling, and there is no impediment to the formation of an inter-liquid band of width $\sim Z t_\perp$. This is consistent with what we would expect from a calculation based upon (Landau) quasiparticles. Formally, the coherence is reflected in the fact that the spectral function $A_{12}(\omega)$ is dominated by the δ-function at $\omega = 0$, indicating that the hopping is almost entirely energy degenerate.

2.2. Luttinger liquids

We now turn to the problem of coupled Luttinger liquids, considering the case of spin-independent electronic interactions, characterized by the anomalous exponent, 2α, of the single-particle Green's function, and charge and spin velocities v_c and v_s. The calculation of $A_{12}(\omega)$ and $A_{21}(\omega)$ is lengthy, and we present only the final results here. Complete details are given in [6]. The *exact* result is

$$\begin{aligned} A_{12}(\omega) &= A_{12}^{\text{low}}(\omega)\,\theta_+[\omega-(v_s-v)\Delta k]\,\theta_+[(v_c-v)\Delta k-\omega] \\ &\quad + A_{12}^{\text{high}}(\omega)\,\theta_+[\omega-(v_c-v)\Delta k] \\ A_{12}^{\text{low}}(\omega) &= \frac{1}{\Gamma(1+4\alpha)}\frac{1}{\Delta v}\left(\frac{a^2}{\bar v\,\Delta v}\right)^{2\alpha}(\omega+(v-v_s)\Delta k)^{4\alpha} \\ A_{12}^{\text{high}}(\omega) &= \frac{1}{(1+2\alpha)}\frac{1}{\Gamma(2\alpha)\Gamma(1+2\alpha)}\frac{1}{\bar v}\left(\frac{a}{2v_c}\right)^{4\alpha} \\ &\quad\times(\omega+(v_c+v)\Delta k)^{2\alpha+1}(\omega-(v_c-v)\Delta k)^{2\alpha-1} \\ &\quad\times {}_2F_1\left(1,1-2\alpha;2+2\alpha;-\left(\frac{\Delta v}{\bar v}\right)\left[\frac{\omega+(v_c+v)\Delta k}{\omega-(v_c-v)\Delta k}\right]\right) \end{aligned} \tag{4}$$

where ${}_2F_1$ is the hypergeometric function, a a short-distance cut-off, and $\bar v \equiv v_c + v_s$, $\Delta v \equiv v_c - v_s$ [8]. The typical morphology of $A_{12}(\omega)$ is shown in figure 1. We observe that $A_{12}(\omega)$ is both non-singular and of wide support, having non-zero weight from just below $\omega = 0$ all the way up to the ultraviolet cut-off. As $\alpha \to 0$, we have $A_{12}(\omega) \to \delta(\omega)$, and one needs to use degenerate perturbation theory to treat the inter-liquid hopping. For $\alpha > 1/2$, $t_\perp$ is a formally irrelevant operator, which is reflected in the fact that the spectral weight in $A_{12}(\omega)$ is pushed to high energies. For $\alpha < 1/2$, but not too small, $A_{12}(\omega)$ is generically 'flat' suggesting that much, if not most, of the hopping occurs via non-degenerate (i.e. 'inelastic') processes. This is reminiscent of situations in more elementary quantum mechanical problems where Fermi's 'Golden Rule' is applied, and clearly raises doubts over any claim that the action of $t_\perp$ is to drive the system to a fixed point in which inter-liquid hopping is coherent.

For simplicity, we shall restrict our discussion from here on to the spinless case, which can be obtained by formally taking $\Delta v \to 0$. The general case will be discussed elsewhere [6].

In calculating $\Gamma_{12}(t)$ it is simplest to consider its time derivative. We find

$$\frac{\mathrm{d}\Gamma_{12}(t)}{\mathrm{d}t} = \frac{t_\perp^2 L}{\pi}\frac{1}{\Gamma(2\alpha)\Gamma(2+2\alpha)}\left(\frac{a}{2v_c}\right)^{4\alpha}\frac{1}{2v_c}\frac{1}{\Gamma(1-2\alpha)}t^{-(1+4\alpha)}$$

$$\times\ \mathrm{Re}\{\mathrm{e}^{\mathrm{i}(v_c-v)\Delta k\,t}[\mathrm{i}\mathrm{e}^{\mathrm{i}\,2\pi\alpha}\Gamma(1-2\alpha)\Gamma(1+4\alpha)\ {}_1F_1(-1-2\alpha,-4\alpha;-\mathrm{i}x) + \frac{1}{2}\frac{(1+2\alpha)}{(1+4\alpha)}\Gamma(2\alpha)\Gamma(1-4\alpha)\ x^{1+4\alpha}\ {}_1F_1(2\alpha,2+4\alpha;-\mathrm{i}x)]\} \tag{5}$$

where ${}_1F_1$ is the confluent hypergeometric function and, for convenience, we have introduced the variable $x = 2v_c\,\Delta k\,t$.

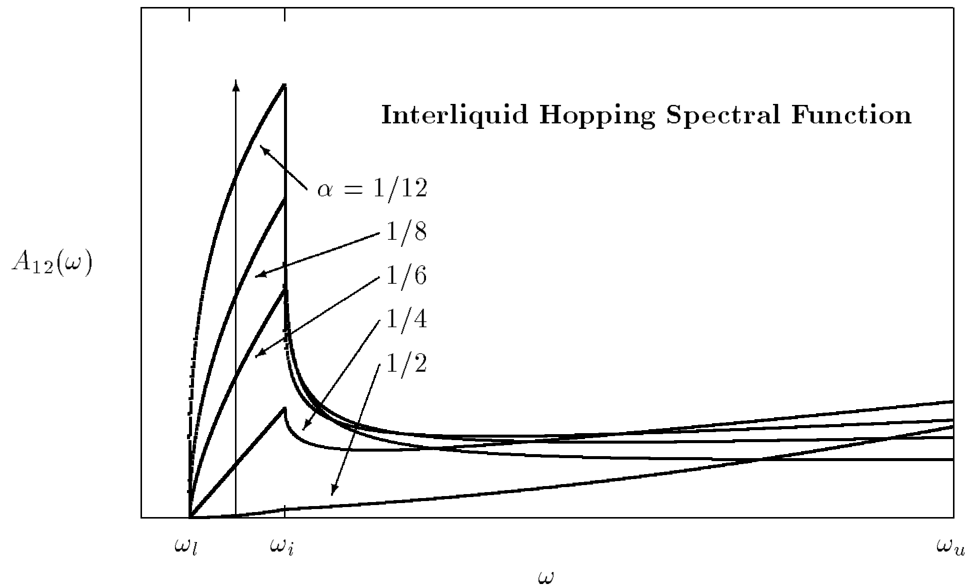

Figure 1. The inter-liquid hopping spectral function for various values of α. Here $\omega_l = (v_s - v)\Delta k$, $\omega_i = (v_c - v)\Delta k$, and ω_u is the ultraviolet cut-off of order v/a. The plots do not include the weak-power-law cut-off-dependent prefactors. The vertical arrow is the $\alpha = 0$ spectral function, $A_{12}(\omega) \propto \delta(\omega)$.

Equation (5) is an *exact* result for the (time derivative of the) inter-liquid hopping rate, to lowest order in $t_\perp$. We use the expansion

$${}_1F_1(a,b;z) = 1 + ab^{-1}z + ab^{-1}(a+1)(b+1)^{-1}z^2/2 + \cdots$$

and, noting that it makes little physical sense to suppose that terms $\mathrm{O}(x^2)$ or higher (i.e. terms of $\mathrm{O}(\Delta k^2)$ or higher) are important in determining the coherence or incoherence of *single-particle* hopping, we retain only the $\mathrm{O}(x^0)$, $\mathrm{O}(x)$ and $\mathrm{O}(x^{1+4\alpha})$ terms. This gives

$$\frac{\mathrm{d}\Gamma_{12}(t)}{\mathrm{d}t} = \frac{t_\perp^2 L}{\pi}\frac{1}{\Gamma(2\alpha)\Gamma(2+2\alpha)}\left(\frac{a}{2v_c}\right)^{4\alpha}\frac{1}{2v_c}\ \cos[(v_c-v)\Delta k\,t]\,t^{-(1+4\alpha)} \times \left\{(1+2\alpha)\left\{-\sin(2\pi\alpha)\frac{\Gamma(1+4\alpha)}{(1+2\alpha)} + \cos(2\pi\alpha)\Gamma(4\alpha)x + \frac{\Gamma(2\alpha)\Gamma(1-4\alpha)}{2(1+4\alpha)\Gamma(1-2\alpha)}x^{1+4\alpha}\right\} - \tan[(v_c-v)\Delta k\,t]\cos(2\pi\alpha)\Gamma(1+4\alpha)\right\}. \tag{6}$$

The latter two terms continuously develop into the correct, coherent result for free fermions as $\alpha \to 0$, and their modification from the Fermi liquid result is closely analogous to the behaviour of the appropriate terms in the derivative of the TLS transition rate upon turning on coupling to an ohmic bath. In the other well understood limit, $\alpha > 1/2$, the entire expression leads to a finite integrated transition probability, $P(t) = \int_0^t \mathrm{d}t' \; \Gamma(t')$, in agreement with the known irrelevance of $t_\perp$ in that limit. In between, the Δk-independent term gives $P(t) \propto t^{1-4\alpha}$, which is long-time convergent and therefore irrelevant if $\alpha > 1/4$, but represents fundamentally incoherent inter-liquid hops if $0 < \alpha < 1/4$. The $\mathrm{O}(\Delta k^{1+4\alpha})$ term requires care to interpret when $\alpha \neq 0$, but we note that the oscillatory prefactor $\cos[(v_c - v)\Delta k\, t]$ will force $\Gamma_{12}(t)$ to be essentially time independent for times $t \gtrsim [(v_c - v)\Delta k]^{-1}$. This effect is analogous to that of non-degeneracy in the TLS [3] where it has been argued to dramatically decrease coherence. In order for lowest-order hopping to be coherent, one must keep to times short enough to avoid the cut-off effect of this prefactor, and the maximum possible Δk for a given time t is $\Delta k^{\max} \sim [(v_c - v)t]^{-1}$. The $\mathrm{O}(\Delta k^{1+4\alpha})$ term in $\mathrm{d}\Gamma/\mathrm{d}t$ is therefore bounded by $\sim \Delta k\, t^{-4\alpha}/(v_c - v)^{4\alpha}$ which has the same form as the term linear in Δk, and we therefore consider only the latter term.

If the term linear in Δk decays slower than t^{-1}, it should be interpreted as a potentially coherent term. For $\alpha > 1/4$ it falls off faster than t^{-1}, and at $\mathrm{O}(t_\perp^2)$ the inter-liquid single-particle hopping is completely incoherent, signalled by the convergence of $\Gamma(t \to \infty)$. *This is despite the relevance of $t_\perp$ in the RG sense for $\alpha < 1/2$.* We therefore expect an incoherent inter-liquid hopping phase for $1/4 < \alpha < 1/2$. There are, however, additional factors enhancing incoherence over and above the time exponent of the $\mathrm{O}(\Delta k)$ term.

First, there is the 'dephasing' prefactor $\cos[(v_c - v)\Delta k\, t]$, analogous to a bias term in a TLS. According to the results from that problem [3], this should enhance incoherence. Further, there are the incoherent processes contributing to the Δk-independent term. For $0 < \alpha < 1/4$ the inter-liquid hopping rate and the integrated transition probability, P, are essentially sums of incoherent and coherent parts, defined by their respective time behaviours. Due to the presence of the dephasing term, the coherent term remains so only for times $t \lesssim [(v_c - v)\Delta k]^{-1}$. As such, $P_{12}^{\mathrm{coh}}(t)$ is bounded above in magnitude by $\sim t_\perp^2 v_c \Lambda^{4\alpha} t^{1-4\alpha}/(v_c - v)$, so

$$\frac{P_{12}^{\mathrm{incoh}}(t)}{P_{12}^{\mathrm{coh}}(t)} \gtrsim \alpha \frac{(v_c - v)}{v_c}.$$

This is *independent* of $t_\perp$, and the purely incoherent channel cannot be eliminated in the $t_\perp \to 0$ limit, as it can in a Fermi liquid. As a result, we are forced to consider the influence of inter-liquid hops upon one another via correlations not automatically included in our $\mathrm{O}(t_\perp^2)$ calculation. To begin with, inter-liquid hops through the coherent channel will be interrupted by the finite probability of a hop through the incoherent channel. Secondly, intra-liquid interactions will lead to scattering of coherent hops by incoherent hops. In the limit $t_\perp \to 0$, the incoherent hops have an arbitrarily long time to scatter the coherent hops (although their density also vanishes in this limit), and we find that the effect of a given incoherent hop on the coherent hops grows at least linearly in time. If it grows faster than linearly, the scattering will be divergent, and hopping should be incoherent as $t_\perp \to 0$ *for any α*.

Combining all of these effects, we expect that as we decrease α from $1/4$, incoherence will be stabilized down to some critical value $\alpha_c < 1/4$ by a combination of the purely incoherent term, the dephasing prefactor which kills coherence if $\Delta k\, t$ is too large, and, in the case of fermions with spin, spin–charge separation, which further suppresses coherence for finite $\Delta k\, t$ [6]. We again emphasize the utility of the spectral function $A_{12}(\omega)$ in indicating the coherent or incoherent nature of the inter-liquid hopping.

3. The approximate single-particle Green's function: calculation and interpretation

We now consider how these same effects might appear in the more conventional calculation of the Green's function for $N \to \infty$ coupled Luttinger liquids of spinless fermions. We will neglect vertex corrections associated with $t_\perp$ and incorporate $t_\perp(k_\perp)$ as an energy-independent self-energy. We are motivated by similar calculations by others [9]; however, we focus on analytic properties of the Green's function not previously treated. Using $G^{-1} = G_0^{-1} - \Sigma$, $G_0(k, \omega) = (v^2k^2 - \omega^2)^\alpha(\omega - vk)^{-1}$, gives

$$G(k, k_\perp, \omega) = \frac{(v^2k^2 - \omega^2)^\alpha}{(\omega - vk) - t_\perp(k_\perp)(v^2k^2 - \omega^2)^\alpha} \tag{7}$$

where we have set the dimensionful high-energy cut-off to 1, and k is momentum along the chains measured from the Fermi surface. Equation (7) must be supplemented by a discussion of the analytic properties of G and G_0, in which we consider only positive k since the case of negative k is essentially identical for $t_\perp(k_\perp) \to -t_\perp(k_\perp)$.

First, recall that the singularities of the Green's function, particularly poles, only have sensible physical interpretations in the second and fourth quadrants of the complex ω-plane. For $k \neq 0$, G_0 has two branch-cut singularities, one for each sign of ω, which must originate in the second and fourth quadrants. Also, G_0 must be real for $-vk < \omega < vk$ since in that region no on-shell decay of an injected fermion is possible. This implies that the phase of G_0 for $\omega > vk$ should be given by $-\alpha\pi$, and by $-\pi - \alpha\pi$ for $\omega < -vk$. Now consider the pole equation, $G_0^{-1}(k, \omega) = t_\perp(k_\perp)$, for $k = 0$ and $t_\perp(k_\perp) > 0$. For $\alpha = 0$, the pole in the complex ω-plane is at $t_\perp(k_\perp)$ and, as we turn on α, it shifts into the fourth quadrant. Moving off the axis into the fourth quadrant, an angle Θ changes the phase of $G_0^{-1}(0, \omega)$ to $\alpha\pi - (1 - 2\alpha)\Theta$, and it is again possible to have a pole if

$$\Theta = \alpha\pi/(1 - 2\alpha). \tag{8}$$

For small α, this pole could be sensibly interpreted as a weakly damped quasiparticle pole, as in a usual Fermi liquid. However, for $\alpha > 1/4$, $\Theta > \pi/2$, the pole enters the fourth quadrant, and the solution has no sensible interpretation as a quasiparticle pole (it would imply an unoccupied, negative-energy quasiparticle state). The last physical solution, which occurred for $\alpha = 1/4$, corresponds to a purely imaginary frequency, entirely in keeping with the idea that $t_\perp$ is acting incoherently at this value of α. For a negative $t_\perp$, an exactly parallel scenario involving the second, instead of the fourth quadrant, results. In both cases, for $\alpha > 1/4$, there is no physically sensible pole resulting from incorporation of $t_\perp$ as a self-energy, and the results are extremely suggestive of incoherence.

The effect is very closely analogous to the behaviour of the Laplace transform of $P(t)$ found in [3] at the onset of incoherence. A similar analogy between the locations of the poles of the single-particle Green's function approximated in this way and the Laplace transform of $P(t)$ in the TLS problem was noted in [10].

We now consider $k \neq 0$. Consider first the case where $t_\perp > 0$. As we move some distance away from the Fermi surface, the singularity at $-vk$ becomes more distant and its effect on the phase less important. For $k^{1-2\alpha} \gg t_\perp$, it becomes possible to circle the singularity at vk without moving appreciably with respect to the singularity at $-vk$, and the phase of $G_0^{-1}(k, \omega)$ close to $\omega = vk$ varies as $\alpha\pi - (1 - \alpha)\Theta$ where Θ is measured downward from the real $\omega > vk$ half-line. As before, at small α the pole has a small imaginary part to its frequency, and it can be sensibly interpreted as a weakly damped quasiparticle pole. However, now α can be as large as $1/2$ before the pole is forced into an unphysical region. Note, however, that for $\alpha > 1/3$, $\Theta > \pi/2$, so the addition of a positive, real self-energy ($t_\perp$) shifts the singularity at vk to a complex energy with a real part *less than* vk. Including spin–charge separation is more complicated, and we state only two of our results for $v_c > v_s$: (1) for $\alpha > 1/6$, the pole lies at an energy whose real part is shifted to below $v_c k$; and (2) for $\alpha > 1/4$, and large k, the pole equation again cannot be satisfied for a physically sensible ω.

Returning to the spinless case, let us follow the pole for $t_\perp < 0$ as we increase k. For $\alpha < 1/4$ this pole lies in the second quadrant, and increasing k eventually pushes it to the imaginary axis. When $k = k_c = v^{-1} t_\perp^{1/(1-2\alpha)} \cos(2\pi\alpha)$, the pole reaches $\omega = \omega_c = \mathrm{i} v k_c \tan(2\pi\alpha)$. Again, the last frequency with a possible physical interpretation is purely imaginary, paralleling what occurred for $k = 0$ and $\alpha = 1/4$.

In addition to this pole, a new pole appears, for negative $t_\perp(k_\perp)$ and $k > 0$, at $\omega \in (-vk, vk)$ given by the real solution of $(vk - \omega)^{1-\alpha}(vk + \omega)^{-\alpha} = |t_\perp(k_\perp)|$. This undamped pole is unphysical, however, in a number of ways. Firstly, G is purely real at the position of the pole only because there is nothing for a fermion at this momentum and energy to decay into in the unperturbed model. It is easy to see, however, that if the pole existed for k close to 0, then there would be accessible decay channels. These are neglected by the omission of vertex corrections. Secondly, for $|t_\perp(k_\perp)| > 2vk$, this pole approaches not $vk - t_\perp$ but $-vk$ (with rapidly vanishing weight) as $\alpha \to 0$. Finally, in a spin–charge-separated Luttinger liquid, and at sufficiently large k, this pole ceases to exist if $\alpha > 1/4$, while for $\alpha < 1/4$ the pole lies just below $v_s k$. In a model with $\alpha < 1/4$ and vanishing spin velocity, e.g. the large-U Hubbard model, the pole is completely dispersionless along the chains. The 'quasiparticles' defined by it have the strange property of a vanishing bandwidth in the direction of large hopping, but a finite bandwidth in the direction of small hopping!

We therefore see that, for $\alpha \geqslant 1/4$ and when analytic properties are treated carefully, this approximate calculation of G gives no indication of the existence of a transversely dispersing quasiparticle. This is in contrast to what has been suggested elsewhere [9]. In fact, if the poles found off the real axis are interpreted as quasiparticle poles and the Fermi surface is identified with the momenta at which the real part of their frequencies cross zero energy, then the conclusion within this approximation is that the Fermi surface warping vanishes completely for $\alpha = 1/4$.

4. Another diagrammatic calculation: Fermi surface warping

Finally, we briefly discuss another diagrammatic calculation addressing coupled Luttinger liquids. For the case of infinitely many coupled chains, the behaviour of $n(k_x, k_y)$ has been studied in lowest-order perturbation theory in $t_\perp$ by Castellani *et al* [7]. They find a shift of $n(k_x, k_y)$ proportional to $\cos(k_y)|k_x^F - k_x|^{-1+4\alpha}$ and interpret this as signalling the instability of the Luttinger liquid. The k-behaviour arises from an integral given in our language by

$$\langle \delta n(k_x, k_y) \rangle \propto \cos(k_y) \int \frac{\mathrm{d}\omega}{2\pi} \, \frac{A^{\Delta N=0}(k_x, \omega)}{\omega} \tag{9}$$

which is *infrared convergent everywhere* for $\alpha > 1/4$.

We have previously argued [1] that the magnitude of the warping of the Fermi surface should be identified with the oscillation frequency of our dynamical calculation, and provides the order parameter for the transition between the phase with 'confined coherence' and the usual phase with coherent transport in all directions. When interpreted in this context, the finding of Castellani *et al* [7], that within perturbation theory the shift in the Fermi occupation function undergoes a qualitative change to convergent behaviour when the hopping is still relevant, supports the notion of incoherence directly.

5. Conclusion

We have calculated exactly the inter-Luttinger-liquid hopping rate to $O(t_\perp^2)$. Of great physical relevance is the effective spectral function for inter-liquid hopping, $A_{12}(\omega)$. We have shown that in a large region of Luttinger liquid parameter space below $\alpha = 1/2$ (the point where $t_\perp$ becomes a marginal operator), $A_{12}(\omega)$ is too broad a function to sustain coherent inter-liquid transport. Single-particle coherence is confined to the one-dimensional chains, in the sense that it is impossible to observe any effects of interference (beyond those observable for completely decoupled chains) between histories which involve inter-liquid hopping. Again, we emphasize that this is *not* the result of an irrelevant $t_\perp$: the *coherence* is confined, but the *electrons* are not.

Our proposal is supported by a careful consideration of the analytic properties of approximate single-particle Green's functions. Even though such approximations are uncontrolled, we find no evidence in these calculations to suggest anything other than that there can exist a phase of relevant, but incoherent, inter-liquid transport. In fact, all of the results presented here indicate that motion of fermions transverse to the chains can be very different for different α (while still in the region of relevant $t_\perp$), and support the idea that the nature of the renormalization group instability of the $t_\perp = 0$ fixed point can also change. This gives further evidence for the existence of a novel fixed point (one of 'confined coherence') in which transport in one or more (but not all) directions is *intrinsically* (*i.e., in a pure system at zero temperature*) incoherent.

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